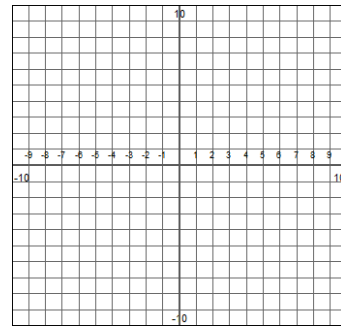
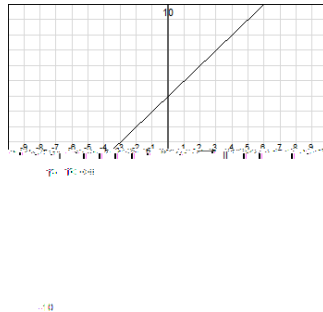
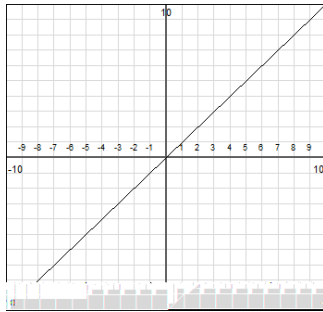


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When we teach linear equations in Algebra1, we teach the simplest linear function (the mother function) as  $y = x$ . We then usually lead to the understanding of the effects of the slope and the y-intercept on the original. Let's focus on the point on the origin.



The shifting that can occur to the point that was on the origin is a combination of a horizontal shift and a vertical shift. We could write the equation of the second graph as  $y = x + 4$  showing the vertical shift as +4 and the horizontal shift as 0. Instead, we write  $y = x + 4$ .

What would the equation be of the linear function that passes through the point in the third graph with slope=1? If the point is the vertical shift in the origin of +6 and the horizontal shift of +2, we could write  $y = x + 6$ , which of course simplifies to  $y = x + 6$ . However, if we did teach  $y = k + m(x - h)$  form, then  $6 = 1(x - 2) + k$  so  $6 = x - 2 + k$  so  $8 = x + k$  so  $k = 8 - x$ .

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Conics (Completing the square to find the standard form of conics):

The general form of a shifted conic is  $Ax^2 + Cy^2 + Dx + Ey + F = 0$  (both A and C  $\neq 0$ ).

If A or C = 0, then the conic is a parabola.

If A and C have the same sign, then the conic is an ellipse. If A=C, then the conic is a circle.

If A and C have opposite signs, then the conic is a hyperbola.

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Graph as before with  $V(5, -2)$

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You try #1: Transform into standard form and determine the conic:

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$$\begin{aligned}2x^2 + 4x + y^2 + 2y &= 6 \\2(x^2 + 2x + 1) + (y^2 + 2y + 1) &= 6 + 2 + 1 \\2(x+1)^2 + (y+1)^2 &= 9 \\ \frac{2(x+1)^2}{9} + \frac{(y+1)^2}{9} &= 1 \\ \frac{(x+1)^2}{\frac{9}{2}} + \frac{(y+1)^2}{9} &= 1\end{aligned}$$

You try #2: Transform into standard form and determine the conic:

$$\begin{aligned}+ \quad - \quad - \quad + \quad &= \\4x^2 - 8x + 9y^2 - 54y &= 49 \\4(x^2 - 2x + 1) + 9(y^2 - 6y + 9) &= 49 + 4 + 81 \\4(x-1)^2 + 9(y-3)^2 &= 36 \\ \frac{4(x-1)^2}{36} + \frac{9(y-3)^2}{36} &= \frac{36}{36} \\ \frac{(x-1)^2}{\frac{9}{2}} + \frac{(y-3)^2}{4} &= 1\end{aligned}$$

Example 3: Transform into standard form and determine the conic:

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You try #3: Transform into standard form and determine the conic:

$$4x^2 + y^2 - 8x + 2y + 8 = 0$$

$$\begin{aligned} & \quad \quad \quad ! \quad \quad \quad ! \\ & \quad \quad \quad ! \quad \quad \quad ! \\ & \quad \quad \quad ! \quad \quad \quad ! \quad \quad \quad ! \end{aligned}$$

This is degenerate so therefore has no solution. The sum of two squares will never be negative.

Example 4: Transform into standard form and determine the conic:

$$9x^2 - y^2 + 18x + 4y + 5 = 0$$

$$\begin{aligned} & + \quad + \quad - \quad - \quad + \quad = - \quad + \quad - \\ & \quad \quad \quad + \quad - \quad ( \quad - \quad ) = \end{aligned}$$

Degenerate, however since it is subtraction, we can attempt to solve.

$$\begin{aligned} 9(x+1)^2 &= (y-2)^2 \\ \pm\sqrt{9(x+1)^2} &= \pm\sqrt{(y-2)^2} \\ \pm 3(x+1) &= y-2 \\ &= 2 \pm 3(x+1) \end{aligned}$$

$$\begin{aligned} & \quad \quad \quad 2 \quad 3(x-1) \\ 2 \quad 3(x-1) & \text{ or } \quad 2-3(x-1) \\ 2 \quad 3 \quad 3 & \text{ or } \quad 2-3 \quad -3 \\ 3 \quad 5 & \text{ or } \quad -3 \quad -1 \end{aligned}$$

This examples graphs out to be two lines.

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